Math 20550 - Summer 2016
Differentiability and the Chain Rule
June 23, 2016

Problem 1. *Is the function* $f(x,y) = xy - 7x^8y^2 + \cos x$ *differentiable at every point of* $\mathbb{R}^2$*? (Recall that* $\mathbb{R}^2$ *is the xy-plane.)*

Problem 2. *Write out the dependency tree for the following case:* $s = f(x,y,z,t)$ *with* $x = x(u,v,w)$, $y = y(v)$, $z = z(u,w)$, *and* $t = t(u,v,w)$.

Problem 3. *Find the gradient of the function* $f(x,y) = x^2 - 2xy + y^2$.

Problem 4. *Find the partial derivatives* $\dfrac{\partial f}{\partial r}$ *and* $\dfrac{\partial f}{\partial \theta}$ *where* $f(x,y) = x^2 - 2xy + y^2$, $x = r + \theta$, *and* $y = r - \theta$ *using the gradient of* f.

Problem 5. *Find the gradient of the function* $f(x,y,z) = ze^{x/z}$.

Problem 6. *Find the derivative* $\dfrac{d}{dt}(f \circ \mathbf{G})(3)$ *where* $f(x,y,z) = ze^{x/y}$ *and* $\mathbf{G}(t) = \langle t-1, t^2-1, t\rangle$.

Problem 7. *Find* f_r *and* f_θ *where* $f(x,y) = x^2y^3$, $x = r\cos\theta$, *and* $y = r\sin\theta$.

Problem 8. *Find* $\dfrac{\partial z}{\partial s}$ *and* $\dfrac{\partial z}{\partial t}$ *where* $z = \tan\left(\dfrac{u}{v}\right)$, $u = 2s + t$, *and* $v = 3s - 2t$.

Problem 9. *Let* $W(s,t) = F(u(s,t), v(s,t))$, *where* F, u, *and* v *are differentiable functions and*

$$\begin{array}{llllllll} u(1,0) = 2 & u_s(1,0) = -2 & u_t(1,0) = 6 & F_u(2,3) = -1 \\ v(1,0) = 3 & v_s(1,0) = 5 & v_t(1,0) = 4 & F_v(2,3) = 10 \end{array}$$

Find $W_s(1,0)$ *and* $W_t(1,0)$.

Problem 10. *Suppose that* z *is implicitly defined as a function of* x *and* y *by the equation*

$$z = e^x \sin(y+z).$$

Find all first partials of z.

Problem 11. *The voltage* V *in a simple electrical circuit is slowly decreasing as the battery wears out. The resistance* R *is slowly increasing as the resistor heats up. Using Ohm's Law,* $V = IR$, *find how the current* I *is changing at the moment when* $R = 400\Omega$, $I = 0.08A$, $\dfrac{dV}{dt} = -0.01V/s$, *and* $\dfrac{dR}{dt} = 0.03\Omega/s$.

Problem 12. *Recall the Ideal Gas Law,* $PV = nRT$, *where* P *is pressure the gas is under,* V *is the volume of the gas,* n *is the number of moles of gas,* R *is the ideal gas constant, and* T *is the temperature of the gas. (The constant* $R = 8.314J/K \cdot mol$, *but you can just leave it as R.)*

The pressure of 1 *mole of an ideal gas is increasing at a rate of* $0.05kPa/s$ *and the temperature is increasing at a rate of* $0.15K/s$. *Find the rate of change of the volume when the pressure is* $20kPa$ *and the temperature is* $320K$.